

✂ Class 12 Mathematics – Chapter: Vectors

1. Introduction

- Vectors are quantities having both magnitude and direction.
 - Used to represent physical quantities like displacement, velocity, force.
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2. Types of Vectors

- Zero Vector: Magnitude zero, no direction.
 - Unit Vector: Magnitude one.
 - Equal Vectors: Same magnitude and direction.
 - Opposite Vectors: Same magnitude, opposite direction.
 - Position Vector: Vector from origin to a point.
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3. Representation of Vectors

- In 2D: $\vec{a} = a_1\hat{i} + a_2\hat{j}$
 $\vec{a} = a_1\hat{i} + a_2\hat{j}$

- In 3D: $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
 $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

4. Operations on Vectors

- **Addition:** $\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$
 $\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$
- **Subtraction:** $\vec{a} - \vec{b} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 - b_3)\hat{k}$
 $\vec{a} - \vec{b} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 - b_3)\hat{k}$
- **Scalar Multiplication:** $k\vec{a} = ka_1\hat{i} + ka_2\hat{j} + ka_3\hat{k}$
 $k\vec{a} = ka_1\hat{i} + ka_2\hat{j} + ka_3\hat{k}$

5. Magnitude of a Vector

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

6. Dot Product (Scalar Product)

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta = a_1 b_1 + a_2 b_2 + a_3 b_3$$

- Properties:

- Commutative: $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- Distributive over addition.

7. Cross Product (Vector Product)

- $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$
- $\hat{n}$ is a unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$.
- Computed using determinant:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

8. Applications

- Physics problems: force, velocity, and acceleration.
 - Geometry: area of parallelogram and triangle using cross product.
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9. Exam Tips

- Practice vector addition and scalar multiplication.
- Memorize formulas for dot and cross product.
- Understand geometric interpretations.
- Solve application problems thoroughly.